# Compact Geometric Structures in Graphics

Norbert Bus

October 8, 2015





## OUTLINE

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## A Compact Structure

Well-Separated Pair Decomposition

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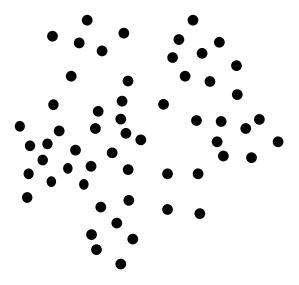
## A Compact Structure

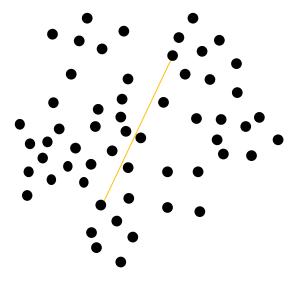
Well-Separated Pair Decomposition

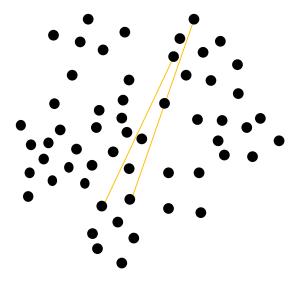
#### Global Illumination

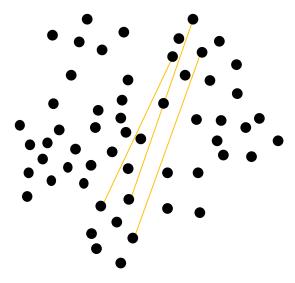
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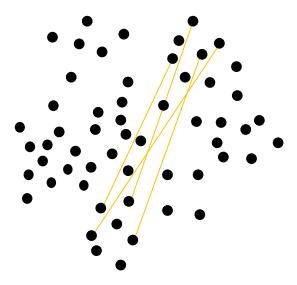
IlluminationCut

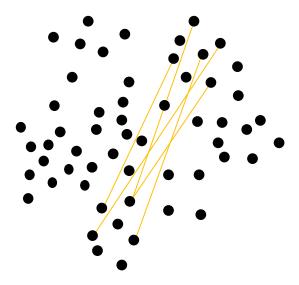


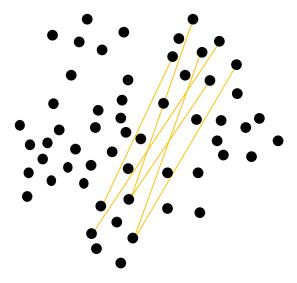




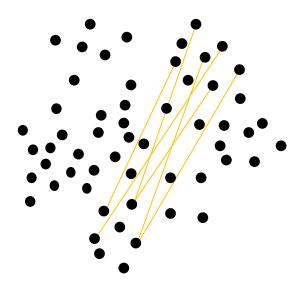


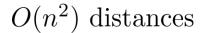




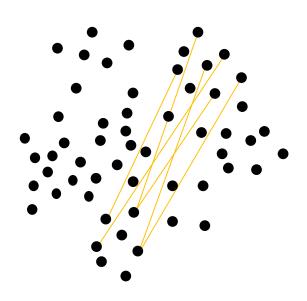


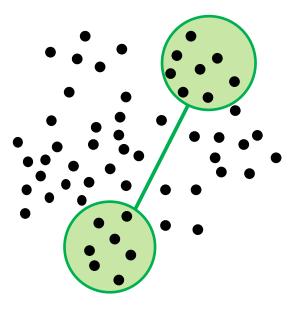
## $O(n^2)$ distances





How to represent them compactly?



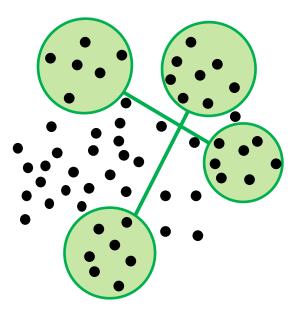


 $\mathcal{P}$ : n points in  $\mathbb{R}^2$ 

 $O(n^2)$  distances

How to represent them compactly?

pairs of clusters

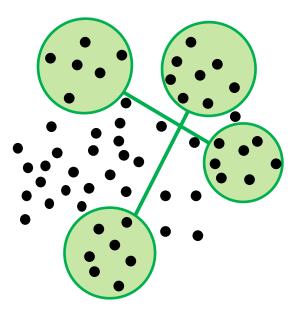


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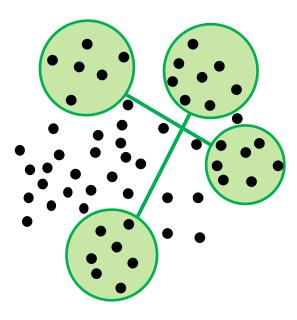


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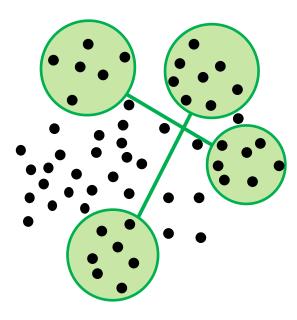
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#### WSPD:

- set of pairs:  $\{(Q,R)|Q,R\subseteq\mathcal{P}\}$ 



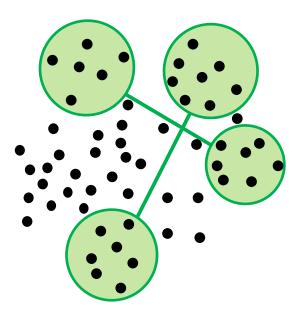
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- set of pairs:  $\{(Q,R)|Q,R\subseteq\mathcal{P}\}$
- every cluster pair is well-separated



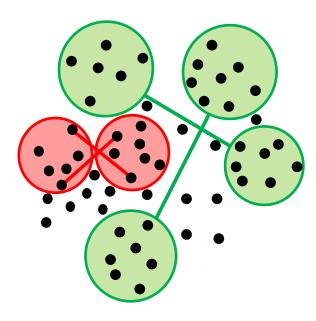
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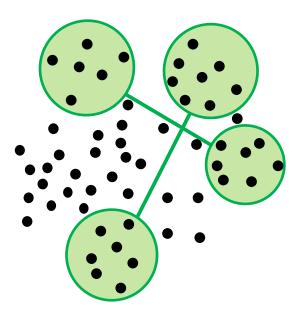
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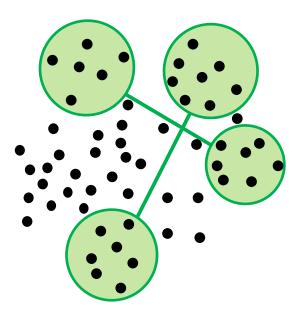
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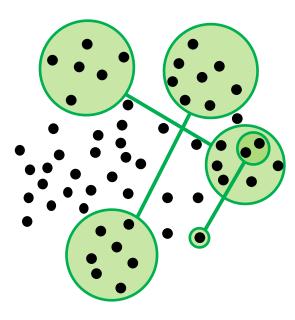
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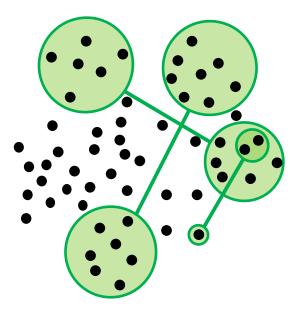
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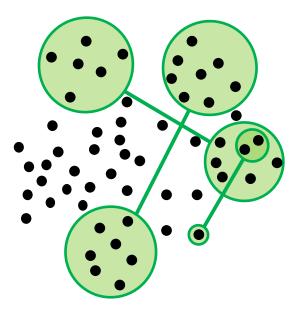
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#### Theorem:



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**Theorem:** O(n) pairs are sufficient

Given: A scene description, comprised of

**Given:** A scene description, comprised of geometry



Given: A scene description, comprised of

geometry

materials





#### Given: A scene description, comprised of

geometry materials lighting







Given: A scene description, comprised of



geometry



materials



Task: Render photorealistic images



Given: A scene description, comprised of



geometry



materials

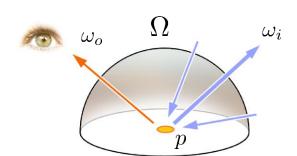
lighting



Task: Render photorealistic images



Solving the rendering equation (simple form):



$$L_o(p,\omega_o) = L_e(p,\omega_o) + \int_{\Omega} f_r(p,\omega_i,\omega_o) L_i(p,\omega_i) (\omega_i \cdot n) d\omega_i$$

[Kajiya 1986]

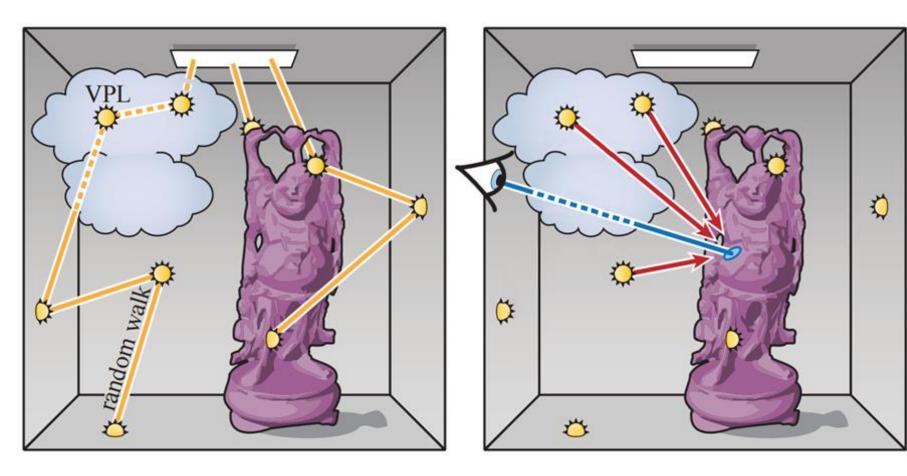
## Many-Lights Methods

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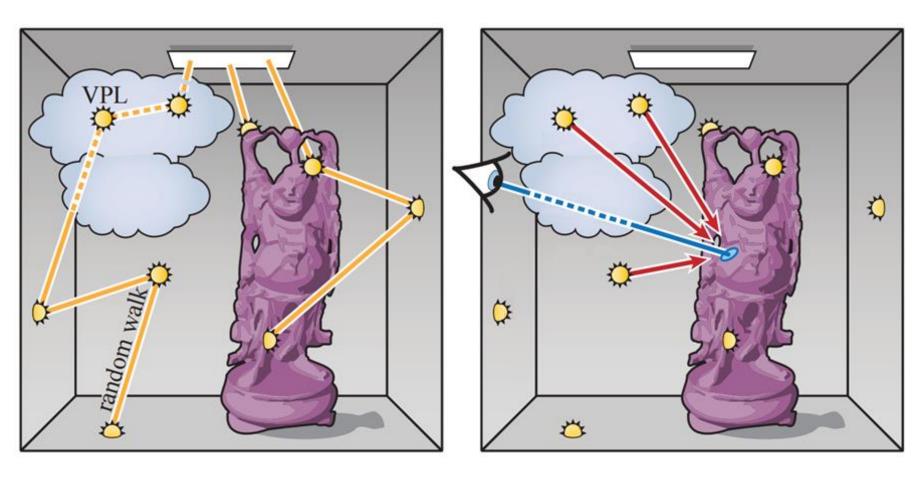
[Keller, SIGGRAPH 1997] Instant Radiosity

## Many-Lights Methods

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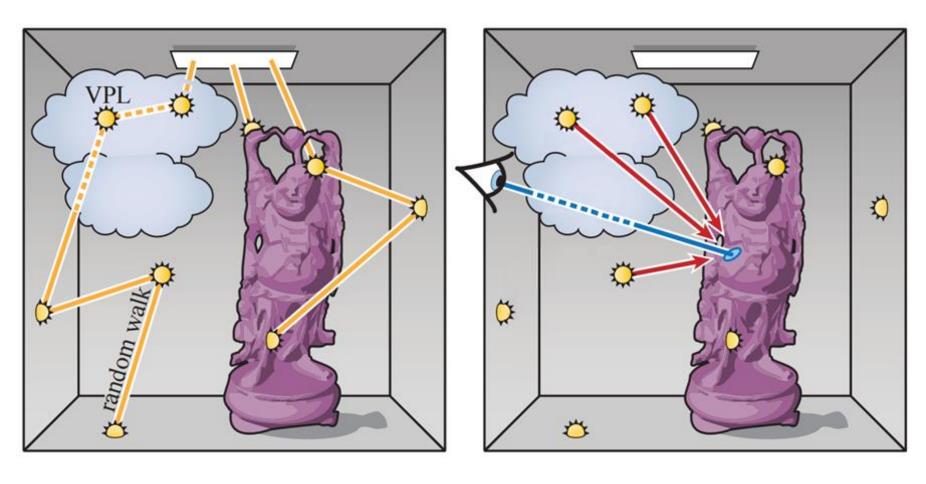


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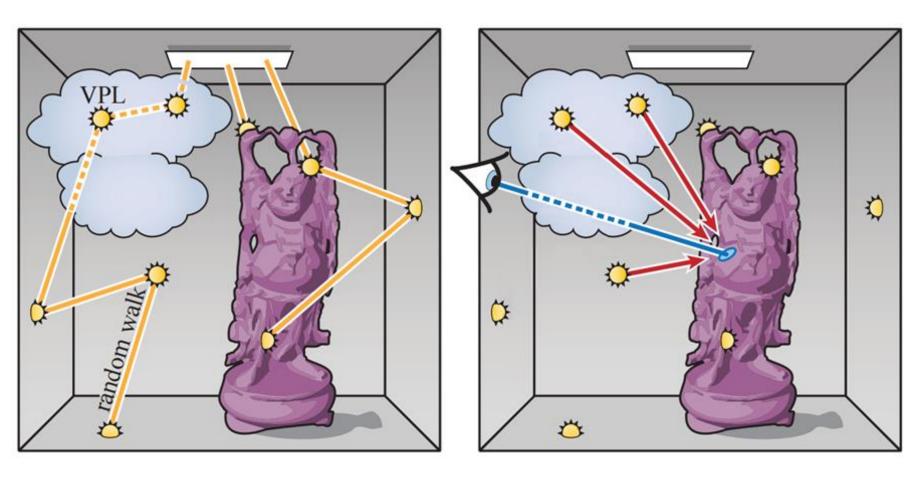
$$L(p,\omega) =$$

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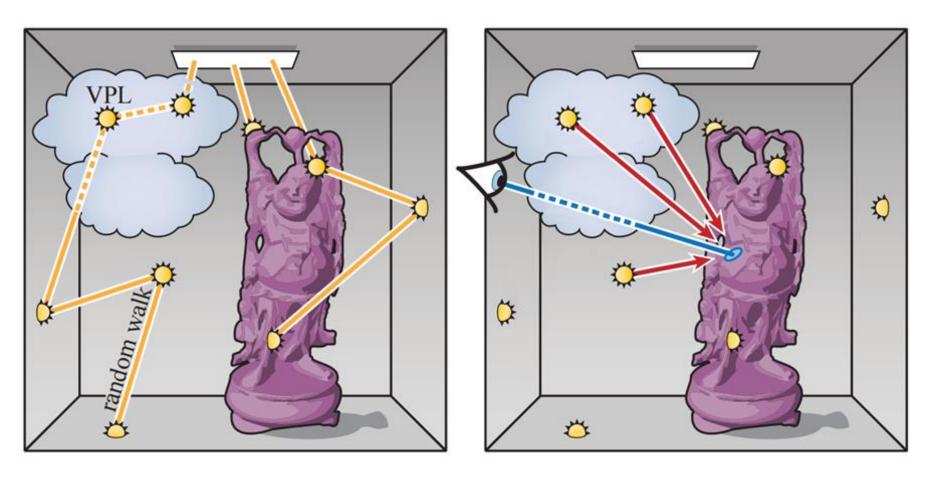
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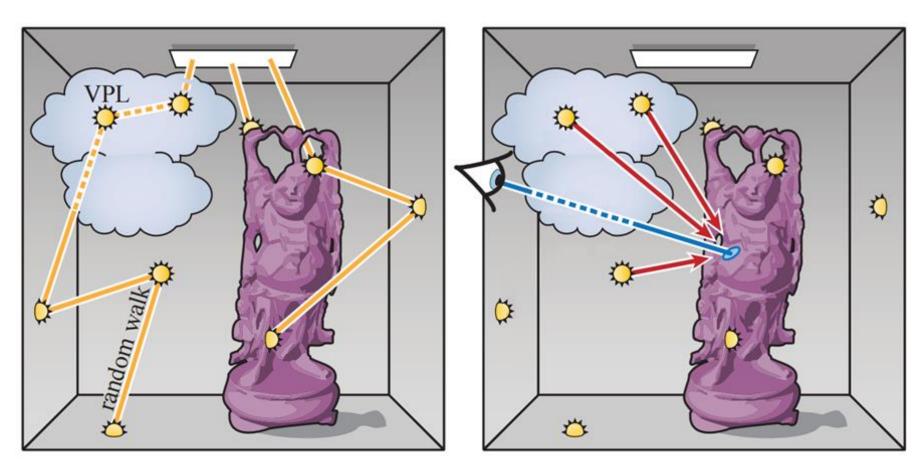
$$L(p,\omega) = \sum_{s \in S} I_s$$

[Keller, SIGGRAPH 1997]



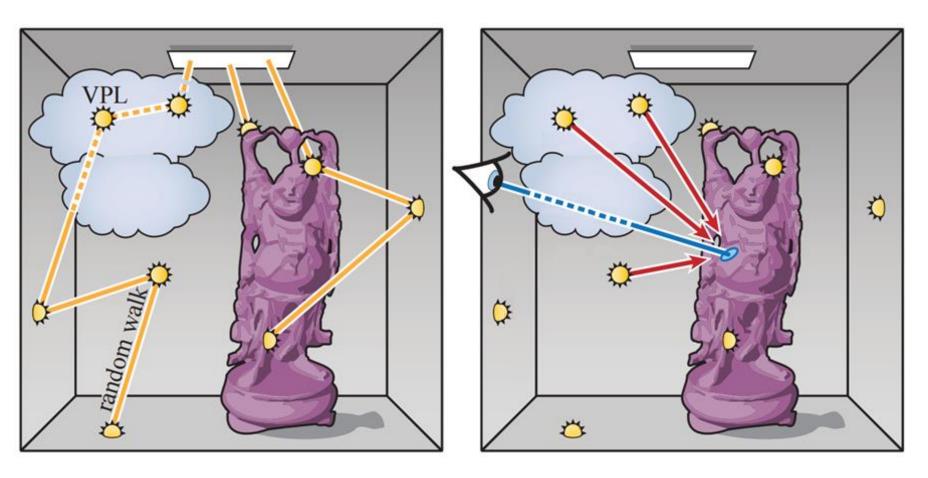
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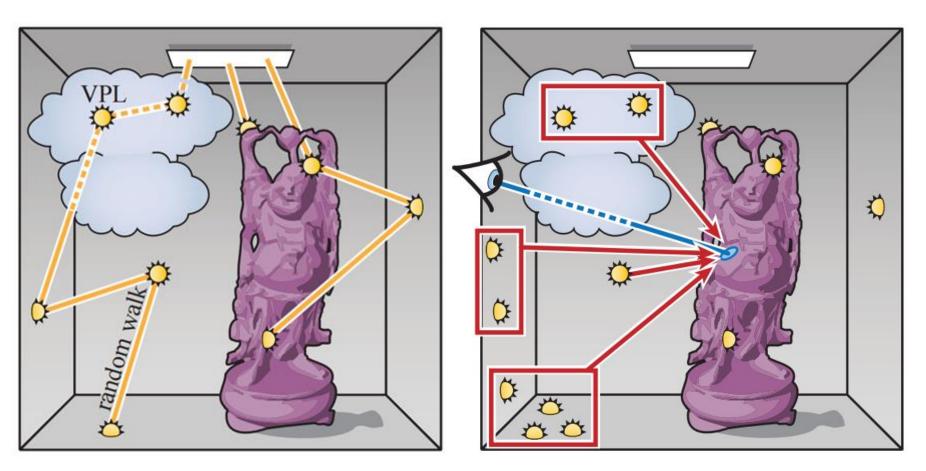
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Millions of VPLs  $\rightarrow$  Cluster lights





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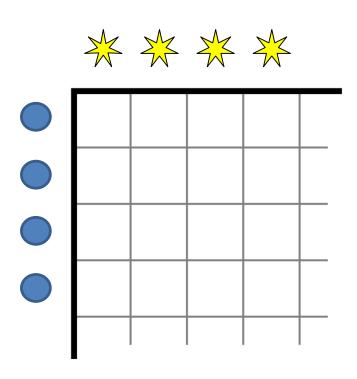
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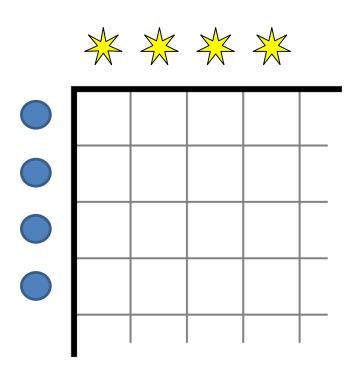
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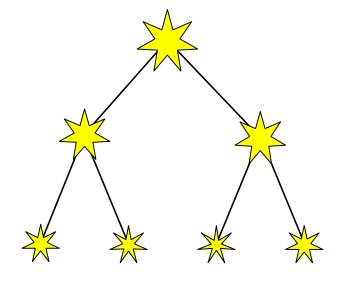
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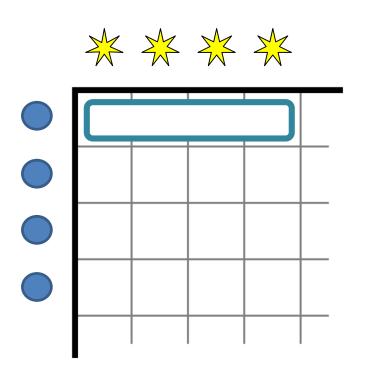
light transport matrix

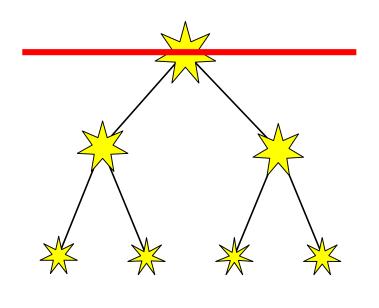




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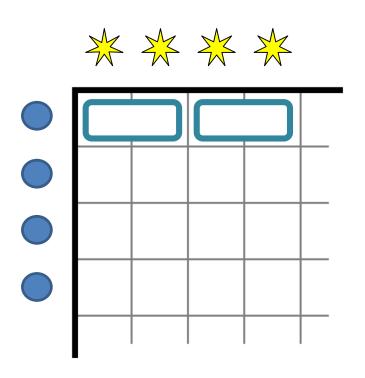
light tree

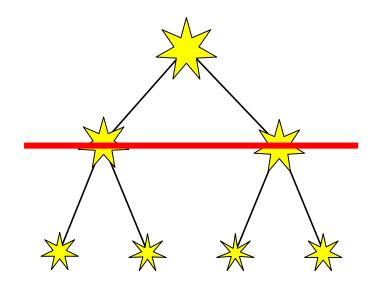




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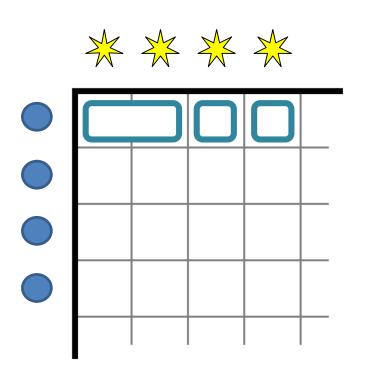
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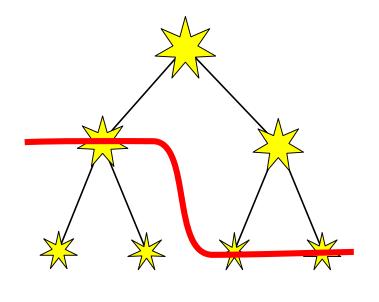




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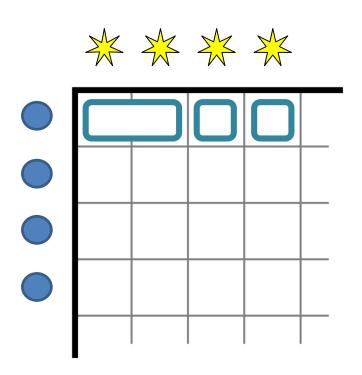
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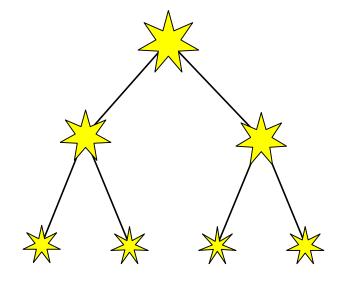




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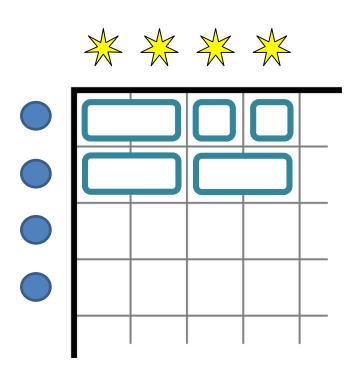
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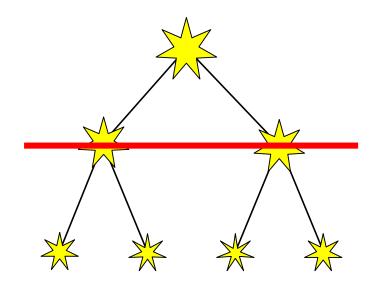




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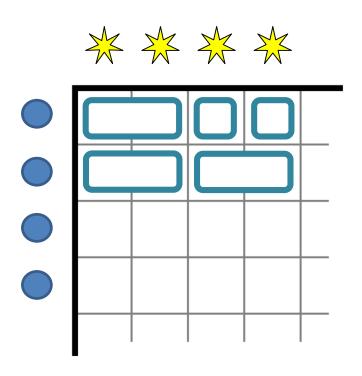
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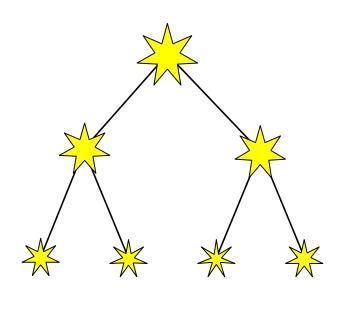


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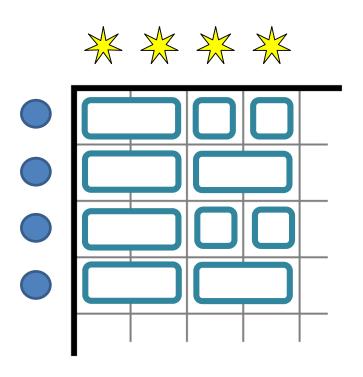
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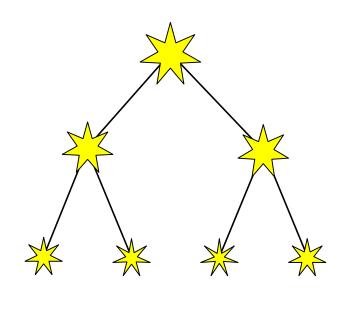




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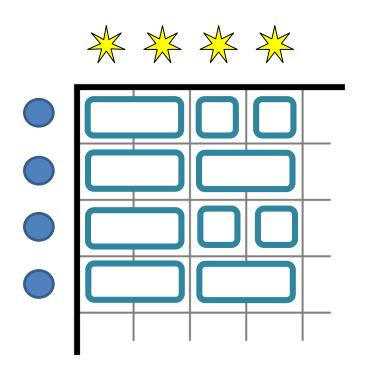


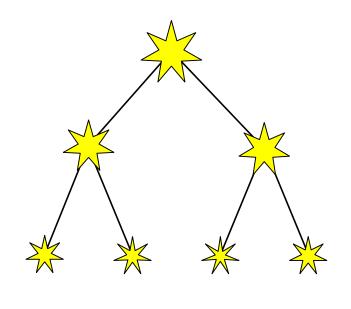




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light transport matrix

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Selecting the cut is still expensive

WSPD:

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In a WSPD each point pair is present in exactly one cluster pair

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Clusters form a clustering for each individual point  $\{Q \mid Q \subset \mathcal{P}, (Q, R) \in WSPD, p \in R\}$ 

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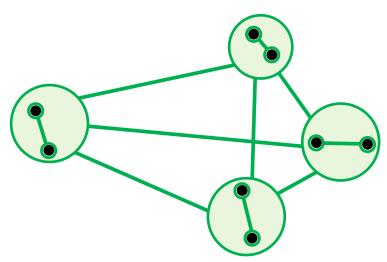
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lacktriangle

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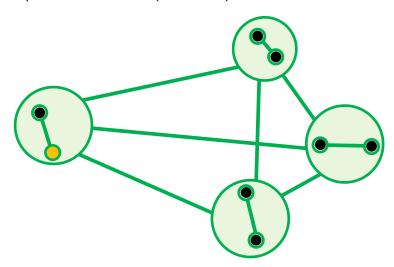
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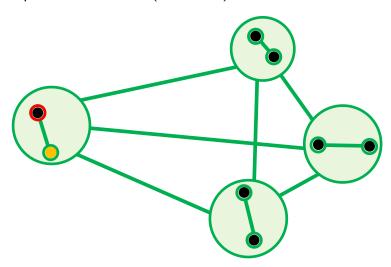
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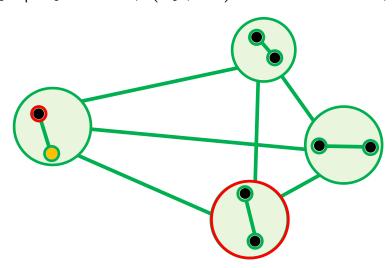
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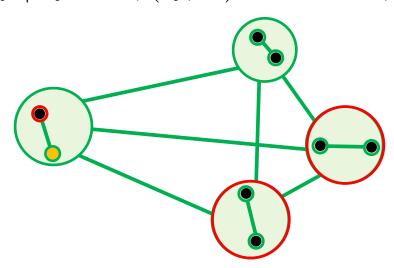
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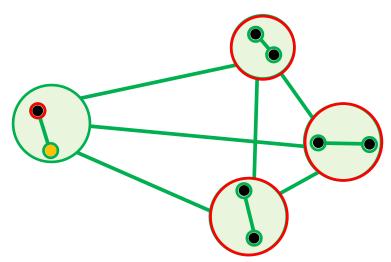
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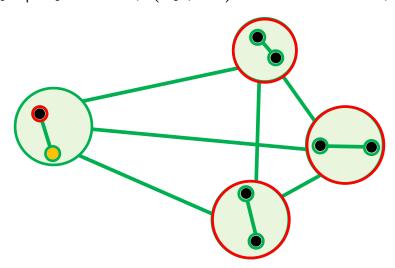


#### WSPD:

In a WSPD each point pair is present in exactly one cluster pair

Clusters form a clustering for each individual point

$$\{Q \mid Q \subset \mathcal{P}, (Q, R) \in WSPD, p \in R\}$$



Use the WSPD to store all clusterings compactly

## Algorithm:

## Algorithm:

Preprocessing phase

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Create WSPD of VPLs – stores clusterings

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Adjust it to be more adapted to illumination

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Rendering

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Pick the closest VPL and take its clustering

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Pick the closest VPL and take its clustering

Only requires provably minor adjustment for each shaded point

## Algorithm:

### Preprocessing phase

Create WSPD of VPLs – stores clusterings

Adjust it to be more adapted to illumination

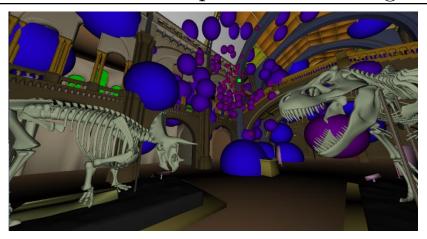
### Rendering

Pick the closest VPL and take its clustering

Only requires provably minor adjustment for each shaded point

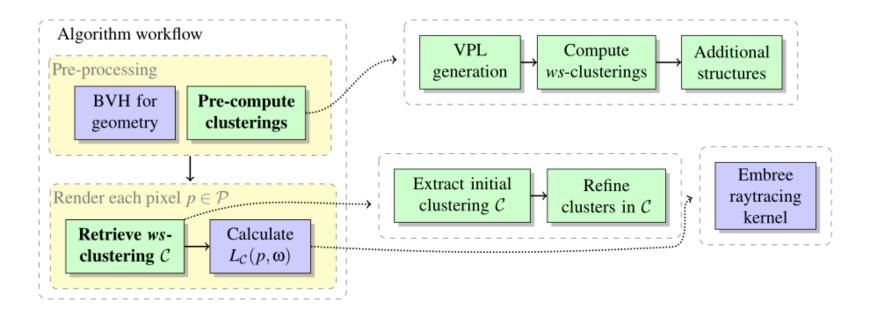
#### Theorem:

Let p be an arbitrary point and s be its nearest neighbor. There is only  $O(\frac{1}{\epsilon^6})$  refinement needed to create a well-separated clustering for p.



## System overview

Easy to integrate into existing framework



Lightcuts



#### WSPD



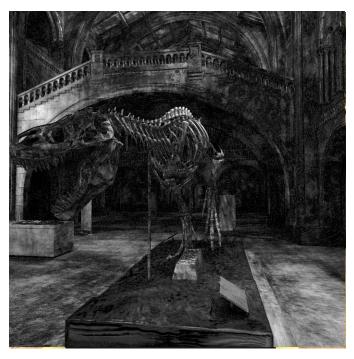
Lightcuts



WSPD



Lightcuts



Time: 515.06 sec RMSE: 0.00467

WSPD



Time:  $190.27 \sec (2.7x)$ 

RMSE: 0.00465

Lightcuts



Time: 515.06 sec RMSE: 0.00467

WSPD



Time:  $190.27 \sec (2.7x)$ 

RMSE: 0.00465

+ very fast rendering

Light cuts



Time: 515.06 sec RMSE: 0.00467

WSPD

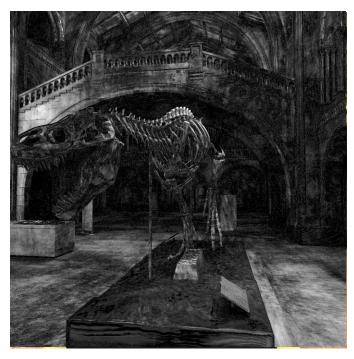


Time:  $190.27 \sec (2.7x)$ 

RMSE: 0.00465

- + very fast rendering
- long preprocessing

Lightcuts



Time: 515.06 sec RMSE: 0.00467

WSPD

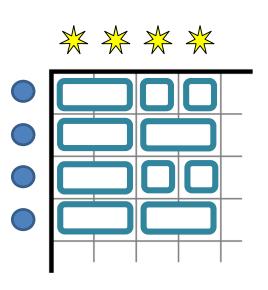


Time:  $190.27 \sec (2.7x)$ 

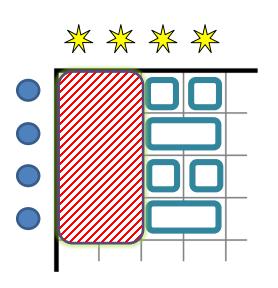
RMSE: 0.00465

- + very fast rendering
- long preprocessing
- diffuse only BRDF

Observe what happens over many pixels

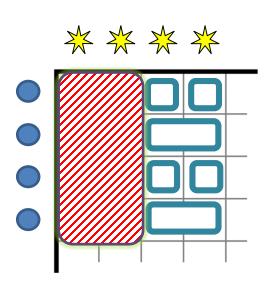


Observe what happens over many pixels



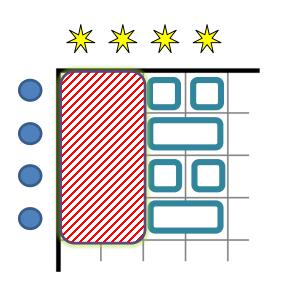
Observe what happens over many pixels

Repeated calculations for the same clusters

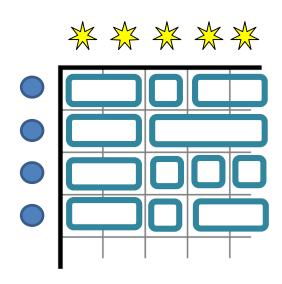


Observe what happens over many pixels

Repeated calculations for the same clusters

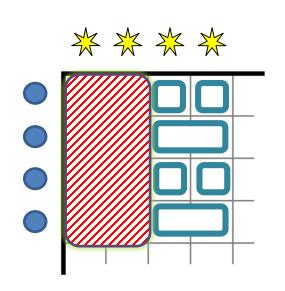


Cluster similar shaded points

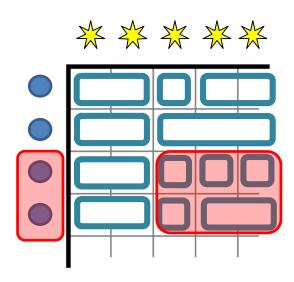


Observe what happens over many pixels

Repeated calculations for the same clusters

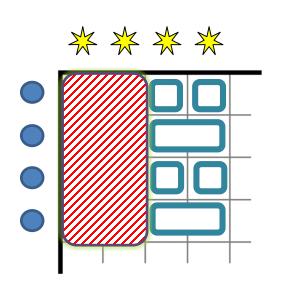


Cluster similar shaded points

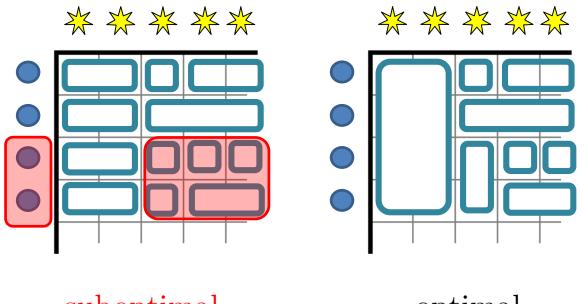


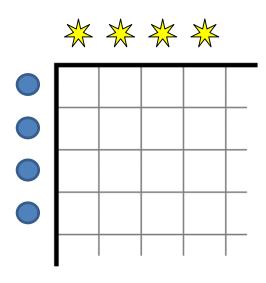
Observe what happens over many pixels

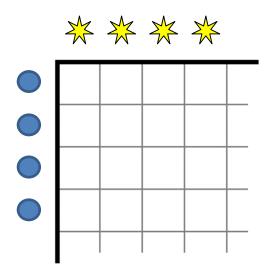
Repeated calculations for the same clusters

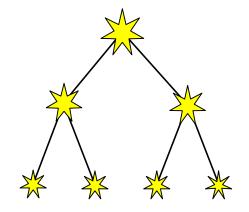


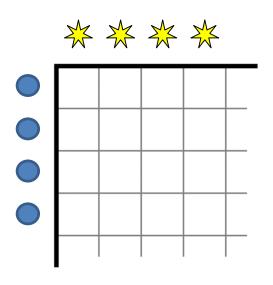
Cluster similar shaded points

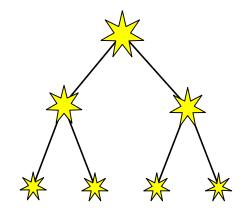




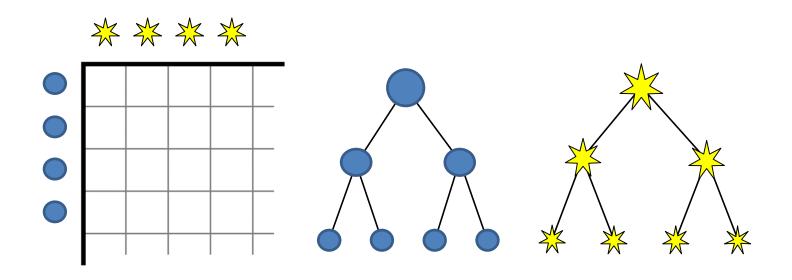




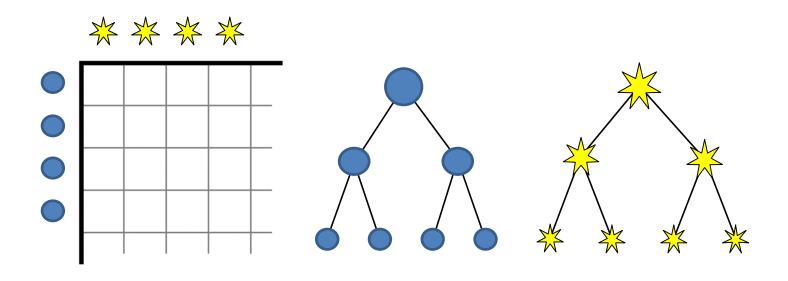




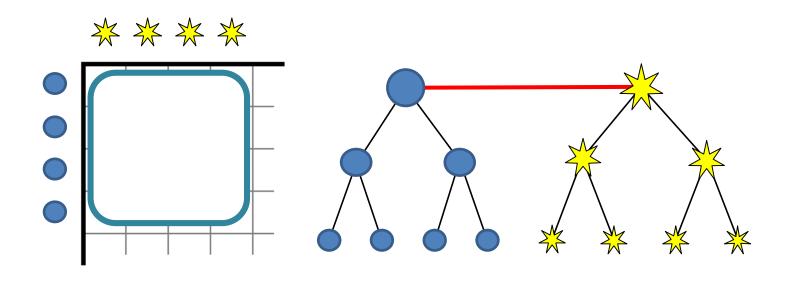
Instead of clustering the points use a hierarchical clustering structure



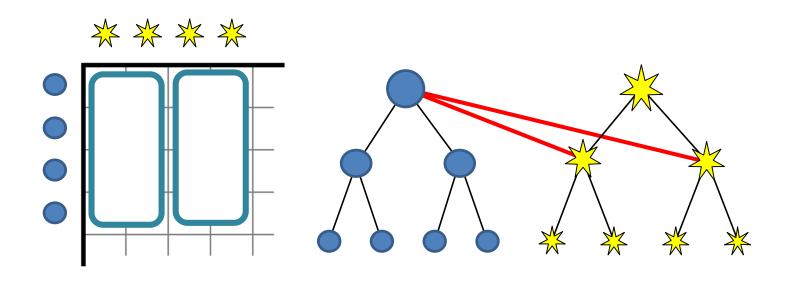
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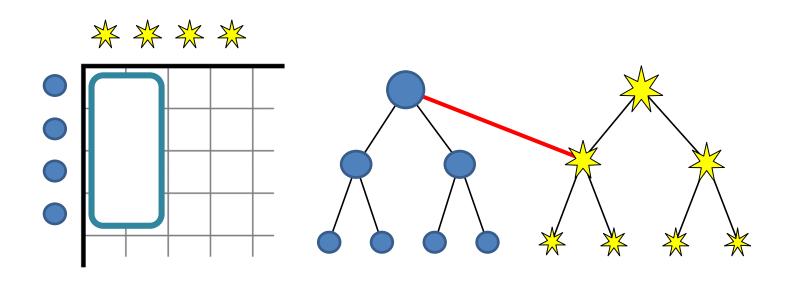
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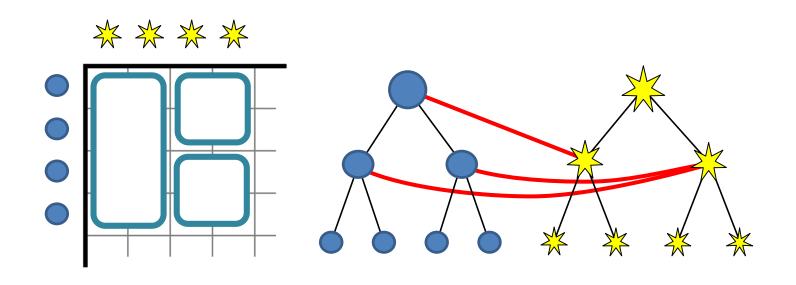
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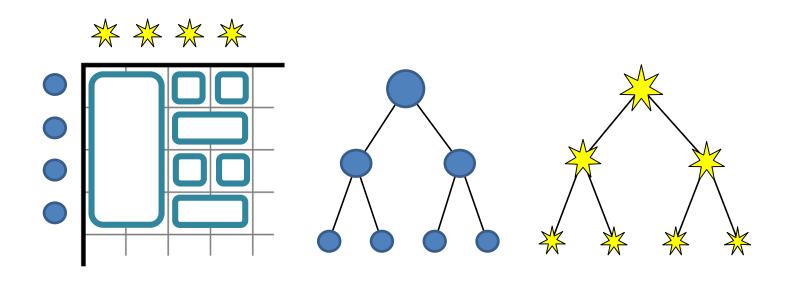
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Instead of clustering the points use a hierarchical clustering structure

# EFFICIENCY

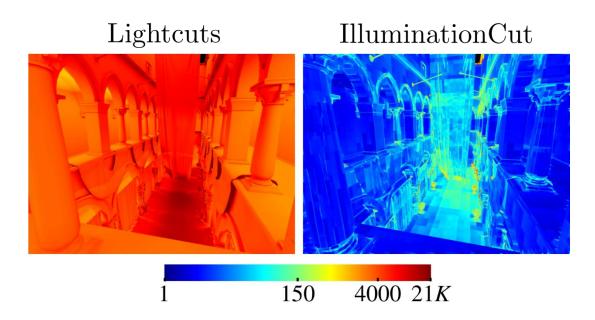
## **Efficiency**

Clustering costs (amortized)

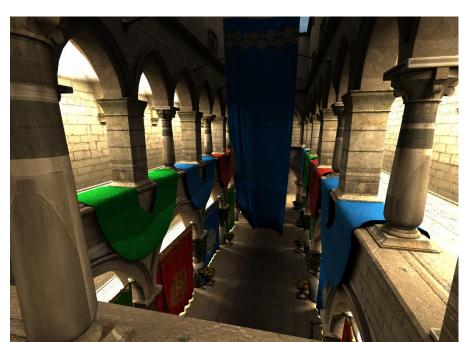
#### **Efficiency**

## Clustering costs (amortized)





Lightcuts



Illumination Cut



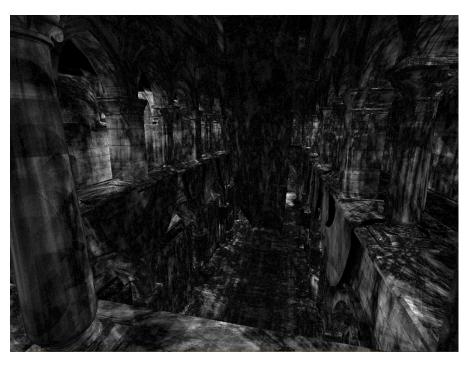
Lightcuts



Illumination Cut



Lightcuts



Time: 233.31 sec RMSE: 0.00591

IlluminationCut



Time:  $71.83 \sec (3.3x)$ 

RMSE: 0.00574

Lightcuts



Illumination Cut



Lightcuts



Illumination Cut



Lightcuts



Time: 183.07 sec RMSE: 0.01033

Illumination Cut



Time:  $43.04 \sec (4.2x)$ 

RMSE: 0.01256

Lightcuts



IlluminationCut



Time: 183.07 sec RMSE: 0.01033 Time:  $43.04 \sec (4.2x)$ 

RMSE: 0.01256

+ very fast rendering

Lightcuts



IlluminationCut



Time: 183.07 sec RMSE: 0.01033 Time:  $43.04 \sec (4.2x)$ 

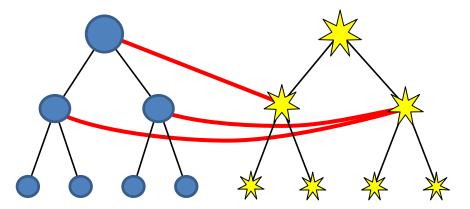
RMSE: 0.01256

+ very fast rendering

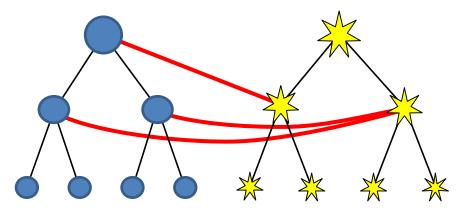
+ specular BRDF

Additional benefit of group pairs:

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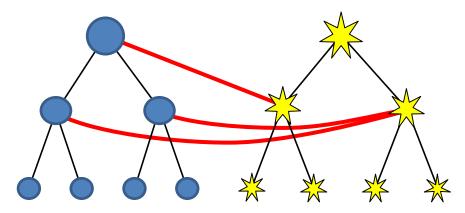


Additional benefit of group pairs:

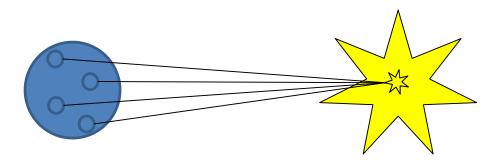


Very similar illumination  $\rightarrow$  we can save shadow rays

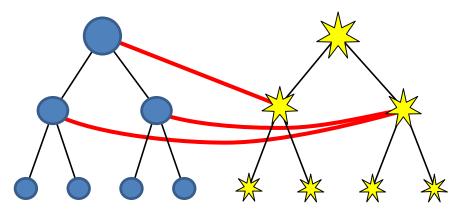
Additional benefit of group pairs:



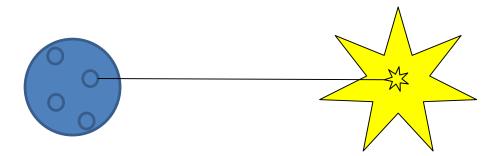
Very similar illumination  $\rightarrow$  we can save shadow rays



Additional benefit of group pairs:



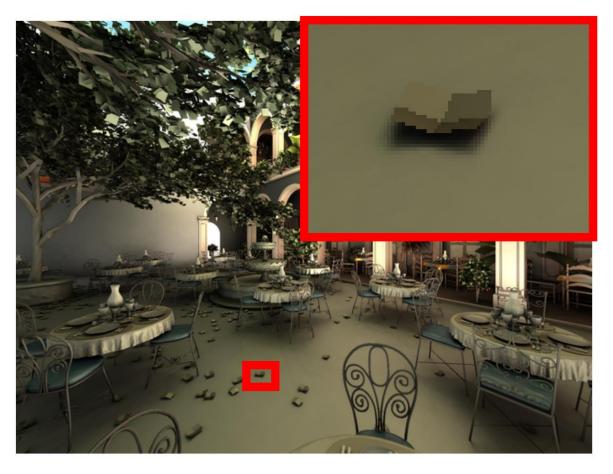
Very similar illumination  $\rightarrow$  we can save shadow rays



#### Reference



#### Reference



#### IlluminationCut – Visibility sampling



IlluminationCut – Visibility sampling



+ even more fast rendering 9.6x speedup

## Published source code

#### Published source code

## Implementation

Most of the state-of-the-art many-lights methods within one framework

Lightcuts

IlluminationCut

LightSlice

Multidimensional Lightcuts

Matrix Row-Column Sampling

Global Illumination Using WSPD

#### Published source code

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Most of the state-of-the-art many-lights methods within one framework

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#### Embree – Intel®

- interactive frame rates with progressive path tracing







# Thank you!